

Name: Mahyar Pirazesh

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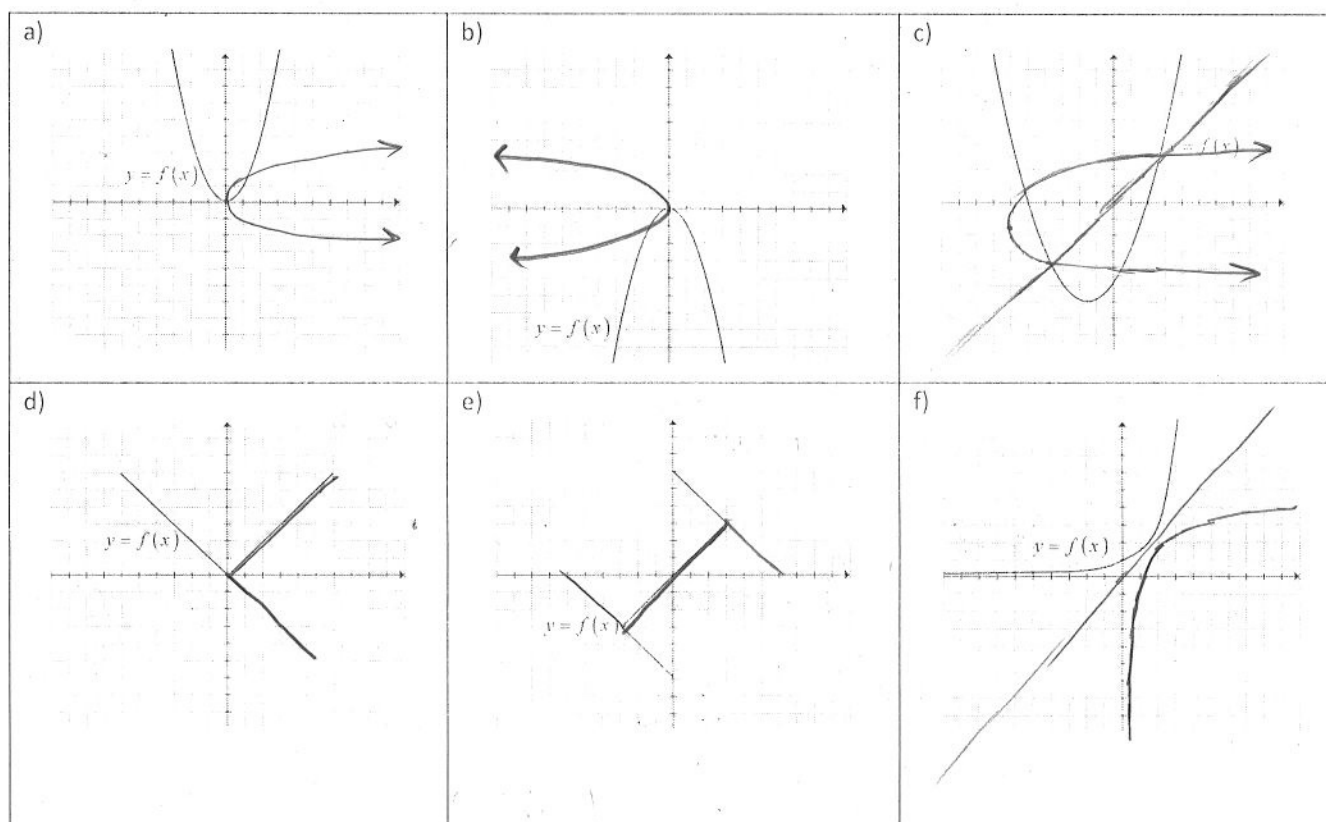
M10 Honours: Section 2.6 Inverse of a Quadratic Functions

1. Given each equation for $y = f(x)$, find the inverse equation $g(x) = f^{-1}(x)$.

a) $y = 3x - 4$ $3x - 4 = y$ $3x = y + 4$ $x = \frac{y+4}{3}$ $\boxed{f^{-1}(x) = \frac{x+4}{3}}$	b) $y = \frac{-8x+11}{2}$ $2y = -8x + 11$ $8x = -2y + 11$ $x = \frac{-2y+11}{8}$ $\boxed{f^{-1}(x) = \frac{-2x+11}{8}}$	c) $y = \frac{2-3x}{4+7x}$ $y(4+7x) = 2-3x$ $4y + 7yx = 2-3x$ $7yx + 3x = 2-4y$ $x = \frac{2-4y}{7y+3}$ $\boxed{f^{-1}(x) = \frac{2-4x}{7x+3}}$
d) $y = \frac{2x-1}{3x+1}$ $y(3x+1) = 2x-1$ $3xy + y = 2x-1$ $3xy - 2x = -y-1$ $x(3y-2) = -y-1$ $x = -\frac{y+1}{3y-2}$ $\boxed{f^{-1}(x) = -\frac{x+1}{3x-2}}$	e) $y = -2x^2; x \geq 0$ $y = -2x^2$ $x^2 = \frac{y}{-2}$ $x = \pm \sqrt{\frac{y}{-2}}$ we want $x \geq 0$, so: $\boxed{g(x) = \sqrt{-\frac{x}{2}} = \frac{\sqrt{-2x}}{2}}$ NOTE: Make sure you always rationalize fraction!	f) $y = 3(x-5)^2; x \geq 5$ $y = 3(x-5)^2$ $\frac{y}{3} = (x-5)^2$ $x-5 = \pm \sqrt{\frac{y}{3}}$ $x = 5 \pm \sqrt{\frac{y}{3}}$ we want $x \geq 5$, so: $\boxed{g(x) = 5 + \sqrt{\frac{x}{3}} = 5 + \frac{\sqrt{3x}}{3}}$
g) $y = (x-3)^2 + 1; x \geq 3$ $y = (x-3)^2 + 1$ $x-3 = \pm \sqrt{y-1}$ $x = 3 \pm \sqrt{y-1}$ since $x \geq 3$: $\boxed{g(x) = 3 + \sqrt{x-1}}$	h) $y = -(x+2)^2 - 5; x \geq -2$ $y = -(x+2)^2 - 5$ $-(x+2)^2 = y+5$ $(x+2)^2 = -y-5$ $x+2 = \pm \sqrt{-y-5}$ $x = -2 \pm \sqrt{-y-5}$ $x \geq -2$ $\boxed{g(x) = -2 + \sqrt{-x-5}}$	i) $y = -3(x+5)^2 + 6; x < -5$ $y = -3(x+5)^2 + 6$ $\frac{y-6}{-3} = (x+5)^2$ $x+5 = \pm \sqrt{\frac{y-6}{-3}} = -5 \pm \frac{\sqrt{18-3y}}{3}$ $x < -5$ $\boxed{g(x) = -5 - \frac{\sqrt{18-3x}}{3}}$

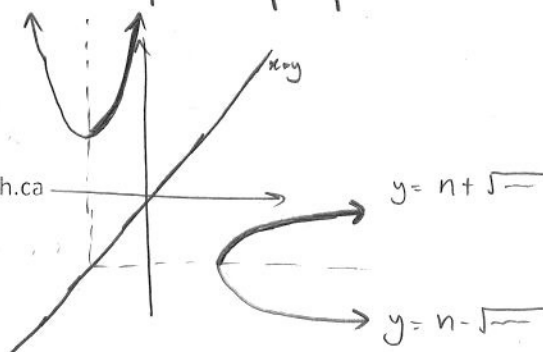
j) $y = 2x^2 - 8x + 11; x \geq 2$ $y = 2x^2 - 8x + 11$ $2x^2 - 8x + 11 - y = 0$ $x = \frac{8 \pm \sqrt{64 - 4(2)(11-y)}}{4} = \frac{8 \pm \sqrt{64 - 88 + 8y}}{4}$ $x = \frac{8 \pm \sqrt{8y - 24}}{4} = \frac{8 \pm \sqrt{8(y-3)}}{4} = \frac{8 \pm 2\sqrt{2y-6}}{4}$ $x = 2 \pm \frac{\sqrt{2y-6}}{2}$ $x \geq 2$ $g(x) = 2 + \frac{\sqrt{2x-6}}{2}$	k) $y = 2x^3 + 6x^2 + 6x + 2$ $y = 2(x^3 + 3x^2 + 3x + 1)$ $y = 2(x+1)^3$ $x = \sqrt[3]{\frac{y}{2}} - 1$ $g(x) = \sqrt[3]{\frac{x}{2}} - 1$	l) $y = 5x^3 - 3x^2 + 6x - 12$ $x = 5y^3 - 3y^2 + 6y - 12$
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2. Graph $y = f^{-1}(x)$ for each function on the same grid. Restrict the domain if necessary:



3. When finding the inverse of a parabola, why do we split the domain along the axis of symmetry? Explain?

because the root function that we get from the inverse of the parabola either opens up or down. The split up point is the A.O.S. The root function opens up if we take the right side of the A.O.S and vice versa.



4. The following points $(3, 5)$, $(-3, -7)$, $(-2, 8)$, $(7, -10)$, and $(-3, -9)$ are on the function $y = f(x)$.

What will the coordinates be on the function: $y = f^{-1}(x)$?

$$(x, y) = (5, 3), (-7, -3), (8, -2), (-10, 7), (-9, -3)$$

5. Under what conditions will both $y = f(x)$ and $y = f^{-1}(x)$ be a function for all values of "x"? Provide examples of such functions:

If $f(x)$ is linear or a cubic function that does not go down and back up again. Ex: $f(x)$ cannot be a vertical or horizontal line



Examples: $y = mx + b$, $y = 5x + 6$, $y = x + 1$, $y = x^3 + x^2 + x$

6. Given the function of the parabola, find the equation, domain, and range of the inverse function.

Note: The domain is restricted so that the inverse is also a function.

i) $y = 2(x-3)^2 + 1; x \geq 3$

$$\frac{y-1}{2} = (x-3)^2$$

$$x = 3 \pm \sqrt{\frac{y-1}{2}} = 3 \pm \frac{\sqrt{2y-2}}{2}$$

$$f^{-1}(x) = 3 + \frac{\sqrt{2x-2}}{2}$$

D: $x \geq 1$
R: $y \geq 3$

iv) $y = 0.2x^2 - 2x + 6; x \geq 5$

$$y = \frac{1}{5}x^2 - 2x + 6$$

$$y = \frac{1}{5}(x^2 - 10x + 30)$$

$$y = \frac{1}{5}(x^2 - 10x + 25) + 1$$

$$y = \frac{1}{5}(x-5)^2 + 1$$

$$\frac{y-1}{\frac{1}{5}} = (x-5)^2$$

$$x = 5 \pm \sqrt{5y-5}$$

$$f^{-1}(x) = 5 + \sqrt{5x-5}$$

D: $x \geq 1$

Range: $y \geq 5$

ii) $y = -3(x+4)^2 - 5; x < -4$

$$y = -3(x+4)^2 - 5$$

$$y+5 = -3(x+4)^2$$

$$(x+4)^2 = \frac{y+5}{-3}$$

$$x = -4 \pm \frac{\sqrt{-3y-15}}{3}$$

$$f^{-1}(x) = -4 - \frac{\sqrt{-3x-15}}{3}$$

D: $x \leq -5$

R: $y < -4$

v) $y = \frac{2}{3}x^2 + 8x + 14; x < -6$

$$y = \frac{2}{3}(x^2 + 12x + 21)$$

$$y = \frac{2}{3}(x^2 + 12x + 36) - 10$$

$$y = \frac{2}{3}(x+6)^2 - 10$$

$$(x+6)^2 = \frac{(y+10)(3)}{2}$$

$$x = -6 \pm \sqrt{\frac{3y+30}{2}}$$

$$x = -6 \pm \frac{\sqrt{6y+60}}{2}$$

$$f^{-1}(x) = -6 - \frac{\sqrt{6x+60}}{2}$$

D: $x \geq -10$

R: $x < -6$

7. If $f(3) = -5$ and $f(-5) = 7$, then what is the value of $|f(-5)| - f^{-1}(-5)$?

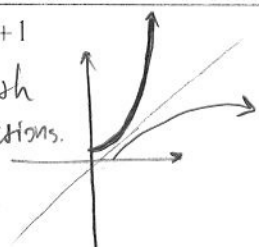
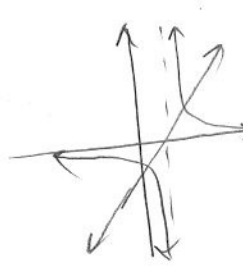
$$f(-5) = 7 \quad f^{-1}(-5) = 3$$

$$|7| - |3| = 4$$

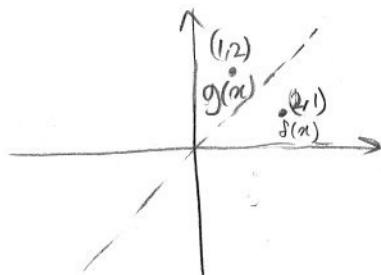
8. At which points on the graph of $y = f(x)$ will it always intersect the inverse function $y = f^{-1}(x)$?

At points where $x=y$

9. Given each of the following functions below, please indicate if both $y = f(x)$ and $y = f^{-1}(x)$ are functions for the domain when $x \in \mathbb{R}$. Justify your answer:

a) $y = 3x + 2$ yes, both $f(x)$ & $f^{-1}(x)$ are functions b/c it is linear.	b) $y = 3(x-3)^2 + 1$ No, only $f(x)$ is a function.
c) $y = 2^x + 1$ Yes, both are functions. 	d) $y = \sqrt{3x-1}$ Yes, both are functions. 
e) $y = \frac{1}{x-3}$ $f^{-1}(x) = \frac{1}{x} + 3$ Yes $xy - 3y = 1$ $x = \frac{1+3y}{y}$	f) $y = x^2 - 3x$ No, $f^{-1}(x)$ is not a function. There are some sets of (x,y) where 2 values of x generate the same y value. This cannot occur in $f(x)$ if we want $f^{-1}(x)$ to be a function. It won't pass the vertical line test. 

10. Given that the $y = f(x)$ is a function, what is the difference between $f(f^{-1}(x))$ vs $f^{-1}(f(x))$?



$$g(f(2)) = g(1) = 2 \quad f^{-1}(x) = g(x)$$

$$f(g(1)) = f(2) = 1$$

$f^{-1}(f(x))$ gives you the x value you plug in
 $f(f^{-1}(x))$ gives back the x value plugged in, but the x value you inputted was on $f^{-1}(x)$, so you end up having the y value from $f(x)$.

Also, if $f(x)$ is a parabola, $f^{-1}(f(x))$ will generate two outputs

11. Suppose $f(x) = (x-3)^2$, what is the value of $f^{-1}(f(4))$? What is the value of $f(f^{-1}(4))$?

$$f(4) = (4-3)^2 = 1$$

$$y = (x-3)^2$$

$$x = 3 \pm \sqrt{y}$$

$$f^{-1}(x) = 3 \pm \sqrt{x}$$

$$f^{-1}(4) = 3 \pm \sqrt{4} = 3 \pm 2$$

$$f^{-1}(1) = 3 \pm 1$$

$$f^{-1}(4) = 5 \quad f(5) = 4$$

$$f^{-1}(1) = 4$$

$$f^{-1}(4) = 1 \quad f(1) = 4$$

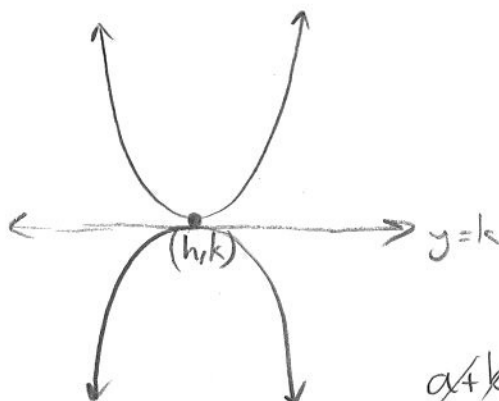
$$f^{-1}(1) = 2$$

$$f^{-1}(f(4)) = 4 \text{ or } 2$$

$$f(f^{-1}(4)) = 4$$

12. The parabola with equation $y = ax^2 + bx + c$ and vertex (h, k) is reflected about the line $y = k$. This results in the parabola with equation $y = dx^2 + ex + f$. What is the value of $a + b + c + d + e + f$?

- a) $2b$ b) $2c$ c) $2a + 2b$ d) $2h$ e) $2k$



$$d = -a$$

$$b = -e$$

$$y = a(x-h)^2 + k$$

$$y = a(x^2 - 2hx + h^2) + k$$

$$y = ax^2 - 2hax + ah^2 + k$$

$$c = ah^2 + k$$

$$y = -a(x-h)^2 + k$$

$$y = -ax^2 + 2hax - ah^2 + k$$

$$f = -ah^2 + k$$

$$a + b + c + d + e + f = 2k$$

13. Challenge: Find the exact values of the equation: $2\sqrt[3]{2x-1} = x^3 + 1$ [Hint: Use Inverse] Adler 2011

$$2\sqrt[3]{2x-1} = x^3 + 1$$

$$\sqrt[3]{2x-1} = \frac{x^3 + 1}{2}$$

To find when $f(x) = f^{-1}(x)$, set

$$x = y$$

$$\frac{x^3 + 1}{2} = x$$

$$x^3 - 2x + 1 = 0$$

$$(x-1)(x^2 + x - 1)$$

$$x_1 = 0 \quad x_2 = \frac{-1 + \sqrt{5}}{2}$$

$$x_3 = \frac{-1 - \sqrt{5}}{2}$$

$$\begin{array}{r} x^2 + x - 1 \\ x-1 \overline{) x^3 - 2x + 1} \\ \underline{-(x^3 - x^2)} \\ x^2 - 2x + 1 \\ \underline{-(x^2 - x)} \\ -x + 1 \end{array}$$

OR

$$x^3 - 2x + 1 + x^2 = x^2$$

$$x^3 + x^2 - 2x - x^2 + 1 = 0$$

$$x^3 + x^2 - 2x + 1 = 0 \Rightarrow x^2(x-1) + (x-1)^2 = 0 \Rightarrow (x^2 + x - 1)(x-1) = 0$$

$$\frac{x^3 + 1}{2} = y$$

$$2y - 1 = x^3$$

$$x = \sqrt[3]{2y-1}$$

$$f^{-1}(x) = \sqrt[3]{2x-1}$$

$$f(x) = \frac{x^3 + 1}{2}$$